Reconsideration of Keynesian Economics and a Generalised Quantity Theory of Money

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§ 1 Introduction

The objects of this paper are, first, to elucidate that Keynes' theory of income determination based on $I=S$ is not compatible with the first postulate of the classical theory of employment accepted by Keynes himself and, second, to generalise the quantity theory of money in a dynamic context by integrating Hicks' so-called "Fixprice Method" of price determination with Keynes' principle of effective demand.

§ 2 The First Postulate and Keynesian Economics

According to Keynes, the classical theory of employment is based on the following two postulates (1936, p. 5).

1 The wage is equal to the marginal product of labour.

2 The utility of the wage when a given volume of labour is employed is equal to the marginal disutility of that amount of employment.

Keynes also explains that the first postulate gives us a "demand schedule" for employment and that the second postulate a "supply schedule" (1936, p. 6). It is the second postulate, among these two, that Keynes rejected as not realistic. In contrast, Keynes was obviously a proponent of the first postulate (1936, p. 17). And Keynes argues that industry is subject to decreasing returns under short-run economic conditions and thus that the rate of real wage has an inverse correlation with an increase in employment.

J. Dunlop (1938) and L. Tarshis (1939) criticised, from an empirical point of view, Keynes' assertion that an increase in employment is inversely correlated with the rate of real wage. They point out that an increase in employment is not necessarily accompanied by a fall in the rate of real wage, according to the time-series data of the United States and the United Kingdom. Although Keynes defends his argument against their criticism (1939), his defense cannot be judged very persuasive.

However, we can think in favor of Keynes about this problem as follows. It is a short-run economy in which capital accumulation and the level of technology are given constant that Keynes presupposed. In this situation the condition of decreasing returns generally prevails, and one can consider the marginal productivity of labour to decrease as employment increases. On the other hand, Dunlop and Tarshis deal with time-series data. Under time-series data, however, it is possible that capital accumulates, technology progresses, and the marginal productivity schedule of labour shifts upward over time. If this is the case, it should come as no surprise whatever that the statistical data show a correlation between an increase in employment and a rise in the rate of real wage.

However, it is not so much a problem in such an *empirical* aspect as the problem in a *theoretical* aspect, i. e., whether the first postulate of the classical theory can be consistent at all with Keynes' principle of effective demand, that is essential. We will set out, in what follows, that the first postulate of the classical theory—leading to the demand schedule for labour—is inconsistent with Keynes' principle of effective demand and that it is necessary for Keynesian economics to part with the first postulate and to replace it with a firms' price-setting principle in a fixprice method as the one, for example, M. Kalecki analysed (1954).

§ 3 Two Theories of Employment in Keynesian Economics

Following the usual notation, we write w=the rate of money wage, p=general price level, Y=real national product, and N=the volume of labour employed. Denoting the production function under short-run economic conditions by $Y=f(N)$, we can assume $f'>0$ and $f''<0$. If we also assume that firms are all under atomistic competitive contitions, and write MRL=marginal revenue of labour and MCL=marginal cost of labour, then employment will be increasing if $MRL>MCL$ and decreasing if $MRL<MCL$. Thus given the current level of p and w, the profit maximisation yields

$$MRL=pf'(N)=w=MCL,$$

which is nothing but that which the first posulate states.

Now it is easy to derive

$$Y=F_s(\underset{(-)}{w},\ \underset{(+)}{p})\cdots\cdots(3.1)$$

from the condition $f'(N)=w/p$. The symbol below each variable represents the sign of the partial derivative with respect to the corresponding argument, and the function F_s is homogeneous of degree zero in w and p. We call this simply "the aggregate supply function". **Fig. 1** describes a possible state of this function. Here w_0 stands for a

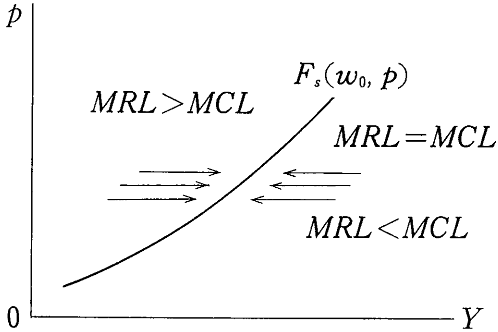

Figure 1.

given rate of money wage. And as is apparent, if the actual economy is situated on this schedule, $MRL=MCL$ holds; whereas employment (and hence output) is increasing with $MRL>MCL$ and decreasing with $MRL<MCL$, as is shown by the direction to which each arrow points. That is, the first postulate of the classical theory demonstrates that output is adjusted in the direction indicated by each arrow as a result of firms' profit-maximising behaviour. Thus we can present, with $t=$ time,

$$\frac{dY}{dt}=\lambda\{F_s(w,\ p)-Y\} \quad \cdots\cdots(3.2)$$

where λ is a postive parameter.

We have so far discussed the essence of the theory of output determination in the case where the first postulate of classical theory is adopted. If w and p are considered to be given in this last equation, we have a dynamic equation concerning Y and can determine Y uniquely. If λ is very large enough (namely, the speed of adjustment in supply of output is high enough), the economy is considered to satisfy the condition of $MRL=MCL$ ceaselessly and this was the case Keynes considered. This, however, obviously disagrees with Keynes' theory of income determination.

In order to see the essence of Keynes' theory of income determination in what follows, we present Keynes' equilibrium system by utilising the following well-known equations:

$$I(r)=S(Y)$$

$$M/p=L(Y,\ r),$$

where I=real investment, S=real saving, M=money supply, L= demand for real money and r=the rate of interest.

Thus this leads to

$$Y=F_d(\underset{(+)}{M},\ \underset{(-)}{p}). \quad \cdots\cdots(3.3)$$

Clearly, this is a homogeneous function of degree zero in p and M. We simply call this "the aggregate demand function". A possible state of this function is drawn in **Fig. 2**, where M_0 represents a given money supply.

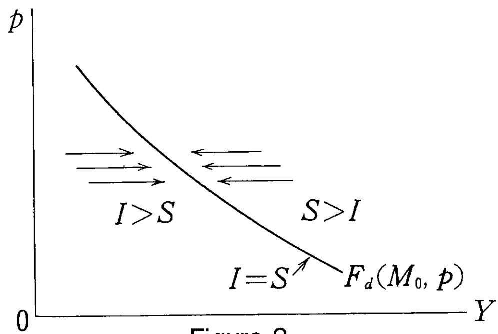

Figure 2.

Now, presupposing $M/p=L$, which will continue in what follows, we can express the dynamic system for income determination in Keynesian economics as

$$\frac{dY}{dt}=\mu\{I(r)-S(Y)\}$$

where μ is a positive parameter. This mechanism is explained by Keynes as follows (1936, p. 184):

> Traditional analysis has been aware that saving depends on income, but it has overlooked the fact that income depends on investment, in such fashion that, when investment changes, income must necessarily change in just that degree which is necessary to make the change in saving equal to the change in investment.

It is easily seen that the above equation can be replaced by the following one which is essentially equivalent to it:

$$\frac{dY}{dt}=\mu\{F_d(M,\ p)-Y\}. \qquad (3.4)$$

Described in **Fig. 2**, this equation demonstrates that, under the current price level, if $I>S$, Y will increase until $I=S$ and that if, on the other hand, $S>I$, then Y will decrease until $I=S$. The horizontal arrows toward the F_d in the figure shows this mechanism, which is the essence of the theory of investment multiplier.

Keynes proposed, on the one hand, the principle of effective demand expressed by the equation (3.4) and supported, on the other hand, the correctness of the first postulate of the classical theory represented by the equation (3.2). These two positions, however, cannot be in agreement with each other. In order to prove this proposition in more detail, let us turn to **Fig. 3** into which Fig. 1 and Fig. 2 are integrated. Two schedules classify four phases. The problem arises in the case where the economy is located in phase ② and ④. Suppose, for example, the economy is situated at point a. Obviously $I<S$ holds at this point, and the market for product lies under excess supply. The firm following Keynes' principle of effective demand will decrease output toward point k. The firm following the first postulate of the classical theory will judge, however, that *MRL* exceeds *MCL* and will increase output toward point c in spite of the presence of excess supply. Thus we may conclude that this **Fig. 3** clarifies straightforwardly that Keynes' theory of investment multiplier is inconsistent with the first postulate of the classical theory of employment and that the two cannot hold true simultaneously. The same sort of disagreement will arise in phase ④ too.

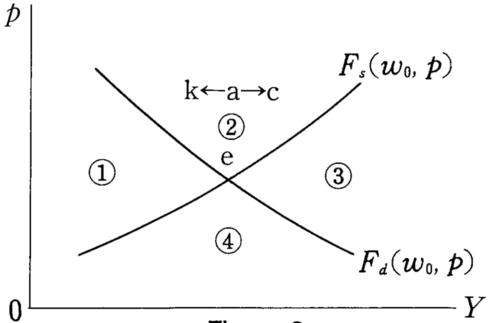

Figure 3.

Since Keynes accepted the rightness of the first postulate of the classical theory, a tremendous number of attempts have been developed to harmonise the first postulate with Keynes' principle of effective demand. However, it is evident from the preceding analysis that such attempts are in vain. The French translator of the *General Theory* states as follows (1979):

> The classical theory of the real wage is not an essential, nor even a necessary, element of the *General Theory*. It can be discarded without any disadvantage. Its elimination would even strengthen the practical conclusions of the *General Theory,* as Keynes pointed out three years after the publication of the work (see Economic Journal, March 1939).

This can be said to represent our position as well. Thus we can conclude that Keynesian economics must eliminate from its theoretical construct the first postulate of classical theory. The first postulate not only disagrees with Keynes' principle of effective demand, but may allow, against Keynes' intention, a possibility that unemployment in Keynesian economics is interpreted as, for example, merely a particular phenomenon appearing in the case where the rate of money wage is fixed in the classical system.

§ 4 Full-Cost Principle and Aggregate Supply Schedule

As was mentioned above, $Y=F_d(M, p)$ in the equation (3.3) represents the level of output in the case where $I=S$ (and $M/p=L$) is satisfied. M can be considered to be an exogenous policy parameter. The problem is how to specify an equation concerning the determination of p. It cannot be denied that Keynes himself presented hardly prospective theories toward this question.

As J. Hicks states, it seems obvious that it is not the price theory based on "Flexprice Method" in which price changes in response to excess demand or supply to ensure the balance between demand and supply, but the one based on "Fixprice Method", in which the causes for price changes exist outside the model, that is consistent with

Keynesian economics (1965; 1974). We should also pay due attention to the contributions of R. Clower (1965), A. Leijonhufvud (1968), R. Barro and H. Grossman (1976), E. Malinvaud (1977) and others in connection with fixprice theory.

However, we will deal with in what follows a price-setting method based on a full-cost principle as one example. This does not mean, of course, that the full-cost principle is the approved best doctrine. We merely adopt this doctrine partly because of its simplicity and partly because of its usefulness. In fact R. Gordon asserted the validity of introducing a full-cost principle into Keynesian economics by discussing as follows (1984, p. 503):

> The full-cost doctine won wide acceptance because its implicit framework of monopoly price-setting was more compatible with the non-market-clearing environment of *The General Theory* than Keynes' own assumption of atomistic competitive firms, and partly because its cost-based procedure of mark-up price determination was consistent with the evidence supporting Means' administered-price hypothesis.

Following the tradition of macroeconomic analysis, let us neglect the cost for intermediate raw materials. Then the price-setting equation based on the full-cost principle can be expressed most simply as

$$p=(1+m)wN/Y,$$

where m=the gross profit mark-up rate, determined by firms, is a given parameter. The ratio of N divided by Y is a labour coefficient in the economy as a whole. If the law of decreasing returns is presupposed, it will increase with an increase in Y in the short run. Many empirical studies indicated, however, that if, even in the short run, the constant returns prevail up to a certain scale of output, then the labour coefficient will remain constant with an increase in Y.

Taking the discussion so far into consideration let us express the above equation as

$$p = G_s(\underset{(+)}{w}, \underset{(+)\text{ or }(0)}{Y}) \quad \cdots\cdots(4.1)$$

which is named "the aggregate supply price schedule". Here one cannot pay too much attention to the fact that this schedule is never the inverse function of the aggregate supply schedule $F_s(w, p)$ in the equation (3.1), which was derived above from the first postulate of classical theory. Indeed the supply-price function in general describes the relationship between supply and market price from the standpoint of a price-taker. The equation (4.1) demonstrates, however, that the firm, under given w and Y, will set price so as to realise its desired profit, and it has nothing to do with the first postulate. And it is also clear that this price-setting equation is quite different from Hicks' Flexprice Method.

Now, if the actual price-setting of firms follows this equation ceaselessly, the system of Keynes' principle of effective demand to be examined will be as follows:

$$\frac{dY}{dt} = \mu\{F_d(M, p) - Y\}$$

$$p = G_s(w, Y).$$

Fig. 4 demonstrates this system. By assumption p is always situated on the aggregate supply-price schedule. Therefore the economy, starting from an arbitrary initial condition, converges to the eqilibrium point e in the direction indicated by arrows along this aggregate supply-price

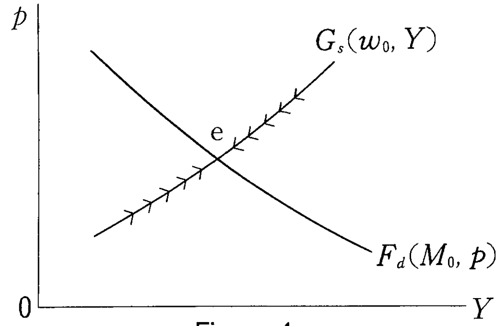

Figure 4.

schedule. Point e is obviously a dynamically stable eqilibrium. In that sense, we can suppose as if the actual economy were located at point e and hence as if Y and p in the actual economy were such that

$$Y = F_d(M,\ p)$$

$$p = G_s(w,\ Y).$$

If M and w were considered exogenous, we can thus determine Y and p uniquely from these two equations.

§ 5 The Variation Equation for Price Level

Now substituting the relationship between r and Y, which is obtained from $I(r) = S(Y)$ to be written as $r = R(Y)$, into $M/p = L$, we have

$$p = \frac{M}{L(Y,\ R(Y))} = G_d(\underset{(+)}{M},\ \underset{(-)}{Y})\ .\cdots\cdots(5.1)$$

This is nothing but the inverse function for $Y = F_d(M,\ p)$ referred to above as the aggregate demand schedule. For obvious reasons, G_d is an increasing function of M and a decreasing function of Y. G_d is called "the aggragate demand price schedule". It is easy to verify that the following holds:

$$\frac{\partial G_d}{\partial M} \div \frac{G_d}{M} = 1. \qquad \cdots\cdots (5.2)$$

That is, if M is changed under given Y, the aggregate demand price schedule will shift upward equiproportionally. The problem is how a change in Y will affect G_d. Then with

$$-\frac{\partial G_d}{\partial Y} \div \frac{G_d}{Y} = \alpha, \qquad \cdots\cdots (5.3)$$

the variation range of α becomes

$$\infty \geqq \alpha > 0. \qquad \cdots\cdots (5.4)$$

Here α becomes infinite in the case where the aggregate demand schedule in **Fig. 2** is vertical. As is evident in Keynesian economics, this case arises either if

(1) the interest elasticity of investment is zero,

or if

(2) the interest elasticity of money demand is infinite.

And in the case where these phenomena are present we say that the economy is suffering from a "Keynesian disease". In any case we can derive from equation (5.1)

$$\frac{\dot{p}}{p} = \frac{\dot{M}}{M} - \alpha \frac{\dot{Y}}{Y}, \qquad \cdots\cdots (5.5)$$

where $dx/dt = \dot{x}$ is used to simplify the symbol.

Next in the above-mentioned aggregate supply price schedule,

$$p = (1+m)wN/Y \equiv G_s(\underset{(+)}{w}, \underset{(+)\text{ or }(0)}{Y}), \qquad \cdots\cdots (5.6)$$

we consider the elasticity of G_s with respect to w. It is obvious that

$$\frac{\partial G_s}{\partial w} \div \frac{G_s}{w} = 1. \qquad \cdots\cdots (5.7)$$

On the other hand if β refers to the elasticity of G_s with respect to Y, namely if we write

$$\frac{\partial G_s}{\partial Y} \div \frac{G_s}{Y} = \beta, \quad \cdots\cdots(5.8)$$

then the variation of β becomes

$$\infty \geqq \beta \geqq 0. \quad \cdots\cdots(5.9)$$

Here β becomes equal to zero in the case where the aggregate supply price schedule is parallel to the horizontal axis, which arises if the constant returns to scale apply to the production function. β becomes infinite, on the other hand, in the case where the economy is under full employment (or of capital), which, as we will mention below, is a phenomenon appearing if the traditional quantity theory of money is prevalent. Here again we can similarly derive from the equation (5.6)

$$\frac{\dot{p}}{p} = \frac{\dot{w}}{w} + \beta \frac{\dot{Y}}{Y}. \quad \cdots\cdots(5.10)$$

We have now rewritten the aggregate demand price schedule and the aggregate supply price schedule, without chaging the essential features of them, into the equation (5.5) and (5.10), respectively, in terms of the rate of change. Then equalising these two equations and arranging terms properly, we can obtain

$$\frac{\dot{p}}{p} = \left\{\frac{\alpha}{\alpha+\beta}\right\} \frac{\dot{w}}{w} + \left\{\frac{\beta}{\alpha+\beta}\right\} \frac{\dot{M}}{M} \quad \cdots\cdots(5.11)$$

and

$$\frac{\dot{Y}}{Y} = \frac{1}{\alpha+\beta} \left\{\frac{\dot{M}}{M} - \frac{\dot{w}}{w}\right\} \quad \cdots\cdots(5.12)$$

Let us first turn to the equation (5.11). This variation equation for the price level demonstrates that the yearly rate of rise (or fall) of the price level is the sum of the following two factors. One is

$$\left\{\frac{\alpha}{\alpha+\beta}\right\}\frac{\dot{w}}{w},$$

which is a cost-push factor, and the other is

$$\left\{\frac{\beta}{\alpha+\beta}\right\}\frac{\dot{M}}{M},$$

a demand-pull factor. One of the weak points of the quantity theory of money is found to concern the fact that it does not consider explicitly the effect of a change in the production cost on the price level. Our variation equation reveals that the price level depends in an essential way not only on M but also on w. In that sense the equation (5.11) can be said to demonstrate the position of a "generalised quantity theory of money" in a dynamic context.

Furthermore, it is noteworthy that the sum of each weight of the rate of change of w and M equal unity. Thus if the rate of money wage rises in an equal proportion with an increase in money supply, the price level will also rise in equal proportion. This conclusion evidently comfirms the validity of what Hicks calls "wage theorem" (1974).

Let us assume for the time being that α is a finite positive parameter. Then if labour is fully emplyed and hence $\beta=\infty$, then

$$\left\{\frac{\beta}{\alpha+\beta}\right\}=1$$

will hold. In this case the price level rises in an equal proportion to money supply. That is,

$$\frac{\dot{p}}{p}=\frac{\dot{M}}{M}$$

is obtained. Indeed, as is apparent from the equation (5.12), Y does not rise despite an increase in M in this case. We have here the case where the quantity theory of money in its purest form prevails. If, on the other hand, $\beta=0$ under the condition of constant returns to scale, then

$$\left\{\frac{\beta}{\alpha+\beta}\right\}=0$$

will hold. In this case an increase in money supply does not affect the price level at all, and since we have

$$\left\{\frac{\alpha}{\alpha+\beta}\right\}=1,$$

the rate of change of the price level equals that of w. Namely,

$$\frac{\dot{p}}{p}=\frac{\dot{w}}{w}$$

is true. We have here the case where cost-push inflation in its purest form applies.

We have so far considered α to be a finite positive parameter. We will assume β to be a finite positive parameter and deal with the case where $\alpha=\infty$ in what follows. As discussed above, this arises when the economy has fallen into a "Keynesian disease". Obviously in this situation

$$\left\{\frac{\beta}{\alpha+\beta}\right\}=0$$

holds. In other words any increase in money supply does not affect the price level in this case. In fact in this case, as is evident from the equation (5.12), any increase in money supply does not affect output, being all absorbed as idle money, nor does any reduction in the rate of money wage. Keynes has argued that when money-increasing policy is not effective as a policy to expand employment, the flexible-wage policy is equally ineffective (1936, p. 266). This corresponds exactly to such a "Keynesian disease" case.

§ 6 Concluding Remarks

We have thus far expressed a possible dynamic system of Keynesian economics based upon the "Fixprice Method" as

$$(\mathrm{K}) \quad \begin{cases} \dfrac{dY}{dt} = \mu\{F_d(M,\ p) - Y\} \\ p = G_s(w,\ Y). \end{cases}$$

And we have elucidated that this (K) system has nothing to do with the first postulate of classical theory. On the other hand if we presuppose, alternatively, the "Flexprice Method" in which market price is so determined as would satisfy the equality between the demand and supply for the current output, and if we consider the output to be determined following the first postulate of classical theory as discussed above, then we can present such a dynamic system as

$$(\mathrm{C}) \quad \begin{cases} \dfrac{dY}{dt} = \lambda\{F_s(w,\ p) - Y\} \\ p = G_d(M,\ Y) \end{cases}$$

This economic model is obviously consistent with classical economics, and the characterisic feature of (C) system in comparision with (K) system would be quite clear.

Of course, (K) system and (C) system both form a dynamically stable economic system. What is more, in the equilibrium state ultimately established, both systems are almost too similar to be identified in terms of the dependence of p and Y on M and w.

However, even though a similarity exists, it provides an important foundation for distinguishing the fundamental property of Keynesian economics from that of classical economics to clarify, as above, the courses which converge to an equilibrium state. In this sense we should not limit our analysis to the equilibrium state only.

We have so far treated the rate of money wage as exogenous. It can be assumed, however, that the rate of rise of money wage rate declines with an increase in the rate of unemployment and that if the rate of unemployment reaches a certain level, it will be zero. In general, however, as is evident from the equation (5.12), output decreases and hence the rate of unemployment tends to rise as the rate of money wage rises. If this is the case, inflation cannot continue to exist with cost-inflation only. Therefore it can be concluded that in order for inflation as an economic phenomenon to be able to continue, it must be accompanied by demand inflation supported by money supply. In this sense inflation may be said to be ultimately a "monetary phenomenon". In order to discuss this problem in detail, however, the rate of change of the money wage rate needs to be made endogenous in relation to the rate of unemployment, as is developed, for example, in the "Phillips curve". But this is a remaining subject.

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