Public Inputs and International Trade:
A Graphical Exposition of the Ricardo-Lindahl Model

Yosiko Yamashige

1 Introduction

There are a number of public inputs which are essential to productive activities. Examples of public inputs are infrastructures (for example, transportation and telecommunication systems, water and energy supply), basic research, police and legal services, education and environment preservation. A common feature of these public inputs is that their benefits are being enjoyed simultaneously by a number of firms. Providing adequate public inputs is an important task for the government to enhance domestic industries' competitiveness in the international market.

In this paper, we provide a graphical exposition of the Pareto-optimal provision of public inputs and its implication on the validity of the conventional trade theorems. Despite the long tradition of the international trade theory, diagramatic approach to examine the comparative static results has not been attempted in the presense of public inputs. Public inputs are incorporated into the Ricardo-Viner model of international trade theory by treating them as intermediate non-traded goods with public good characteristics. It should be noted at the outset that trade-theoretic results discussed in our paper are not new. The model and the notion of equilibrium in our paper can be found in Khan (1983) which shows that, with proper modifications, the basic theorems of international trade still hold in the presence of public inputs. This paper complements the analysis of the Ricardo-Lindahl model in Khan (1983). Namely, by using a graphical exposition, we can explicitly show the changes in the level of public inputs which turns out to play a crucial role in determining the Rybczynski effect and the Stolper-Samuelson effect.

The organization of the paper is as follows. Section 2 presents the model and defines the Lindahl equilibrium. Section 3 discusses the Pareto optimality of the Lindahl equilibrium by introducing the diagram. In Section 4, we turn to the analysis of international trade.

2 The Ricardo-Lindahl Model

An economy is endowed with one primary factor labor, $\mathcal{L}$. Two final goods, indexed by f and m, and a public intermediate good, g, are produced in the economy. The economy is assumed to be too small to influence the international prices, so that the final goods are traded at the exogenously given international prices, p_f and p_m. Public inputs are non-traded intermediate goods; hence, the marginal cost of public input, q, as well as the level of public input are going to be determined within the model. We defer the discussion of the determination of the level of public inputs until the following subsections.

The technology available to the economy is summarized by the following production functions:

$$X_f = F_f(L_f, G) \tag{1}$$

$$X_m = F_m(L_m, G) \tag{2}$$

$$X_g = \frac{1}{a_g} L_g \tag{3}$$

where X_i denotes the output level for sector i, L_i and G denote the amounts of labor and public input devoted to the production in industry i, $(i = f, m, g)$. Public inputs are produced using labor as the only input according to a

Ricardian technology to serve as an intermediate good for the production of final goods. Labor requirement for a unit output of public input is denoted by a_g. The production functions for the final goods, F_f and F_m, are assumed to be continuously differentiable and exhibit constant returns to scale.[1)]

Assuming full employment of the factors of production, the material balance equations are given by the following equations:

$$L_f + L_m + L_g = \mathcal{L} \tag{4}$$

$$G = X_g \tag{5}$$

The problem of a competitive producer in sector i $(I = f, m)$ is to maximize his profits by choosing L_i taking p_i's, the wage rate, w, and the supply of public input G as given. The allocation of labor is thus determined through the marginal productivity pricing;

$$w = p_f F_{fL} = p_m F_{mL} \tag{6}$$

where $F_{iL} = \frac{\partial F_i}{\partial L_i}$. The government produces public inputs efficiently. Therefore, the marginal productivity pricing of labor is also observed here.

$$w = q\frac{1}{a_g} \tag{7}$$

1) The degrees of returns to scale for production functions in which a public input is used as an input captures the degrees of 'publicness' of the public inputs. Mead (1952) distinguished two special types of public inputs. He calls a public input an 'atmosphere' type if the final goods' production functions exhibit constant returns to scale with respect to private inputs alone, i.e., outputs can be doubled by doubling the amount of private inputs, while keeping the level of public input constant. The aggregate production functions exhibit increasing returns to scale. When public inputs are pure public goods, they fall into this type. As the benefits of public inputs become more rival and more excludable due to congestion or limited availability, the degree of returns to scale for final goods' production diminishes. When final goods' production functions exhibit constant returns to scale with respect to all inputs, the externalities associated with public inputs is what Meade calls the 'unpaid factors of production' type.

where q is the marginal cost of public input.

In the Lindahl equilibrium, personalized prices are imposed according to the marginal benefits each producer gains from the use of public inputs. Let q_i denote the personalized prices (or Lindahl taxes) for sector i, then we can write

$$q_i = p_i F_{iG} \tag{8}$$

$$q_f + q_m = w a_g \tag{9}$$

These correspond to equations (2.1) in Khan (1983). Equation (9) can be seen as the government's budget constraint.

Note that Equation (8) and (9) can be obtained by the following government's maximization problem under the competitive labor market.

$$\max_G \; p_f F_f(L_f^*, G) + p_m F_m(L_m^*, G) \; s.t. \; L_f^* + L_m^* + a_g G = L$$

where L_i^* denotes the equilibrium allocation of labor which satisfies the marginal productivity condition (6). The government maximizes the international value of GNP, $p_f X_f + p_m X_m$, with respect to G. By setting the Lagrangean multiplier on the resource constraint to be equal to w, the first-order condition implies

$$p_f F_{fG} + p_m F_{mG} - a_g w = 0$$

$$q_f + q_m - a_g w = 0$$

where $F_{iG} = \frac{\partial F_i}{\partial G}$.

The Lindahl equilibrium is characterized by the following definition.

Definition 1 *A Lindahl equilibrium is constituted by a tuple* (w^*, q^*, q_f^*, q_m^*) *and by a strictly positive tuple* $((X_f^*, L_f^*, G^*), (X_m^*, L_m^*, G^*), (X_g^*, L_g^*))$ *such that*

(i) L_i^* *maximizes* $p_i F_i(L_i, G^*) - w^* L_i$ $(i = f, m)$, *(ii)* L_g^* *maximizes* $q^*\frac{1}{a_g} - w^*L_g$,

(iii) $X_i^* = F_i(L_i^*, G^*)$ $(i = f, m)$,

(iv) $X_g^* = \frac{1}{a_g}L_g^*$,

(v) $L_f^* + L_m^* + L_g^* = \mathcal{L}$,

(vi) $G^* = X_g^*$,

(vii) $q_i^* = p_iF_{iG}^*$, $(i = f, m)$

(viii) $q_f^* + q_m^* = w^*a_g$

3 Optimal Provision of Public Inputs

In this section we graphically illustrate the Lindahl equilibrium defined in the previous section.[2] We will see that the use of the diagram is convenient for analyzing the Pareto optimality condition for public input provision.

Extending the work of Samuelson (1954), Kaizuka (1968) and Sandmo (1975) derived the Pareto optimal condition for providing public inputs assuming constant returns to scale technology. The condition indicates the sum of the marginal rate of substitution between public inputs and private inputs is equal to the marginal cost of public input. Since the condition is analogous to the one in Samuelson (1954) for public consumption goods, we call it the Samuelson-Kaizuka-Sandmo condition. In the context of our model, the Samuelson-Kaizuka-Sandmo condition can be written as:

$$\frac{F_{fG}}{F_{fL}} + \frac{F_{mG}}{F_{mL}} = a_g \tag{10}$$

where $F_{iG} = \frac{\partial F_i}{\partial G}$.

In the following subsection, we show graphically that Equation (10)

2) The diagram is a slight modification of the diagram for illustrating an exchange economy with public consumption goods. See for example Malinvaud (1971). The following diagramatic alaysis is based on Yukutake (1996, Ch.2).

implies the Pareto optimal condition pertaining to public input provision. Then, we depict the Lindahl equilibrium and show its Pareto-optimality. As a preliminary to the analysis of international trade, properties of the Pareto optimal allocation are also examined in section 3.2.

3.1 Lindahl Equilibrium and the Samuelson-Kaizuka-Sandmo Condition

The following equilateral triangle is an analogue of the Edgeworth box diagram. The vertical distance from point E to each side of the triangle measures the amount of labor allocated to each sector; the vertical distances from point E to side AB, AC and BC represent the amount of labor allocated to sector f, m and g respectively. Let the height of the triangle be $\mathcal{L}$. Since the sum of all distances is always equal to the height of the triangle, any point within the triangle represents an allocation, i.e., it always satisfies the resource constraint for labor, $L_f + L_m + L_g = \mathcal{L}$.

The isoquants for output f can be drawn on a skewed plane $BA - BC$ taking B as the origin. Using (3), $L_g = a_g G$, G can be projected onto the axis BA and L_f onto BC. Bf represents $\frac{2}{\sqrt{3}} L_f$ and Bg represents $\frac{2}{\sqrt{3}} a_g G$.[3] Since these projections are proportional to L_f and G, we can simply transfer the familiar isoquants drawn in the orthogonal plane to the skewed plane $BA - BC$. The isoquants drawn in the skewed plane preserve the following properties; they are convex to the origin B and the contour corresponds to a higher output level as it moves away from the origin. (See Figure 1.)

With regard to the 'slope' of the isoquant, we need a more careful interpretation.[4] It is useful to introduce new but familiar coordinates, x and y, to measure the slopes. (See Figure 1.) The x and y coordinates of point E on the

3) $Bf = \sin(60^\circ) L_f$ and $Bg = \sin(60^\circ) a_g G$.

4) The following treatment of the 'slope' of the isoquant is due to Okuno and Suzu-

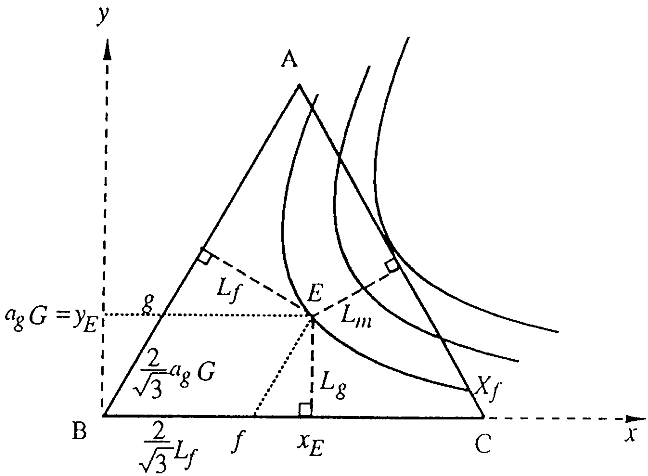

Figure 1: Allocation of Labor and Isoquants

isoquant X_f are

$$x_E = \frac{2}{\sqrt{3}} L_f + \frac{1}{\sqrt{3}} a_g G \tag{11}$$

$$y_E = a_g G \tag{12}$$

Since point E is on the isoquant

$$\overline{X}_f = F_f(\frac{\sqrt{3}}{2} x_E - \frac{y_E}{2}, \frac{y_E}{a_g})$$

On totally differentiating, we obtain

$$d\overline{X}_f = 0 = F_{fL}(\frac{\sqrt{3}}{2} dx_E - \frac{1}{2} dy_E) + F_{fG} \frac{1}{a_g} dy_E$$

Hence the slope of the isoquant can be written as

$$\left.\frac{dy_E}{dx_E}\right|_f = \frac{1}{-\frac{2}{a_g\sqrt{3}}\frac{F_{fG}}{F_{fL}} + \frac{1}{\sqrt{3}}} \tag{13}$$

Similarly, the isoquants for commodity m can be drawn on the $CA - CB$

mura (1988).

plane taking C as the origin. Using (11) and (12), and substituting $L_m = \mathcal{L} - a_g G - L_f$, the isoquant for commodity m satisfies

$$\overline{X}_m = F_m(\mathcal{L} - y_E - \frac{\sqrt{3}}{2}x_E + \frac{y_E}{2}, \frac{y_E}{a_g})$$

By totally differentiating, we obtain

$$d\overline{X}_m = 0 = F_{m\mathcal{L}}(-\frac{\sqrt{3}}{2}dx_E - \frac{1}{2}dy_E) + F_{mG}\frac{1}{a_g}dy_E$$

which implies the slope of the isoquant to be

$$\left.\frac{dy_E}{dx_E}\right|_m = \frac{1}{\frac{2}{a_g\sqrt{3}}\frac{F_{mG}}{F_{m\mathcal{L}}} - \frac{1}{\sqrt{3}}} \tag{14}$$

The advantage of using the triangle is that it explicitly brings out the public good characteristics of G. At E, the same amount of G is employed in both sectors.

Next, we depict efficient allocations in the diagram. Pareto optimality implies the Samuelson-Kaizuka-Sandmo condition to hold. The marginal rates of substitution between labor and public input are no longer required to be equalized between sectors, hence the usual box diagram is unsuitable to depict the efficient allocations because tangencies between the isoquants for each sector does not imply efficiency. The triangle diagram, however, still allows us to depict the efficient allocations by the locus of all the 'tangencies' between the isoquants of the two sectors. The line PO in Figure 2 depicts the efficiency locus. The properties of the efficiency locus is discussed in the following subsection.

We can establish the following result.

Lemma 1 *The Samuelson-Kaizuka-Sandmo condition is satisfied if and only if* $\left.\frac{dy_E}{dx_E}\right|_f = \left.\frac{dy_E}{dx_E}\right|_m$.

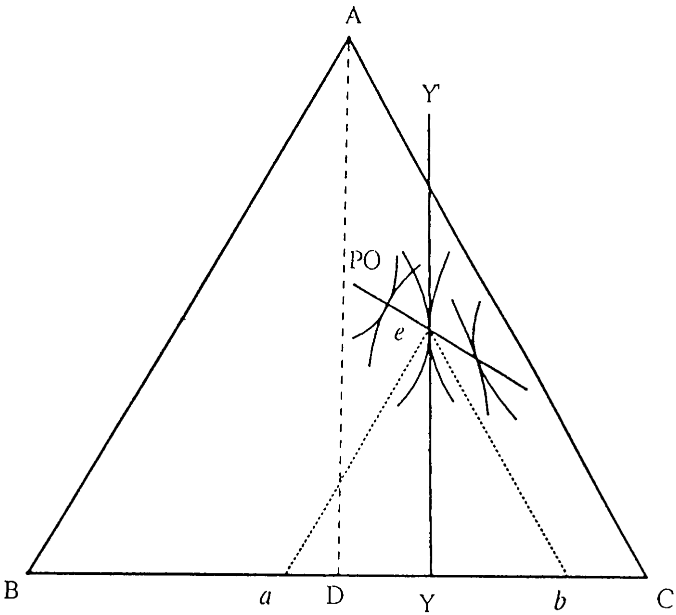

Figure 2: Efficient Allocations

Proof.

i) Along the efficiency locus, the isoquants for both sectors are tangent to each other, i.e., $\frac{dy}{dx}|_f = \frac{dy}{dx}|_m$. Using (13) and (14), we have

$$-\frac{2}{a_g\sqrt{3}}\frac{F_{fG}}{F_{fL}}+\frac{1}{a_g\sqrt{3}}=\frac{2}{a_g\sqrt{3}}\frac{F_{mG}}{F_{mL}}-\frac{1}{a_g\sqrt{3}}$$

A simple calculation yields the Samuelson-Kaizuka-Sandmo condition;

$$\frac{F_{fG}}{F_{fL}}+\frac{F_{mG}}{F_{mL}}=a_g$$

ii) Suppose the Samuelson-Kaizuka-Sandmo condition is satisfied. We can rewrite the condition as

$$\frac{F_{mG}}{F_{mL}} = a_g - \frac{F_{fG}}{F_{fL}}$$

Substituting above expression into (14) yields

$$\left.\frac{dy}{dx}\right|_m = \frac{1}{\frac{2}{a_g\sqrt{3}}(a_g - \frac{F_{fG}}{F_{fL}}) - \frac{1}{\sqrt{3}}} = \frac{1}{-\frac{2}{a_g\sqrt{3}}\frac{F_{fG}}{F_{fL}} + \frac{1}{\sqrt{3}}} = \left.\frac{dy}{dx}\right|_f$$

We have $\frac{dy}{dx}|_f = \frac{dy}{dx}|_m$. ∎

We are now ready to depict the Lindahl equilibrium by introducing the cost line YY'. The line YY represents the cost line $p_f\overline{X}_f = wL_f + q_fG$, whose slope in terms of x-y coordinates is, by using (11) and (12)

$$\left.\frac{dy_E}{dx_E}\right|_{YY}^{f} = \frac{1}{-\frac{2}{a_g\sqrt{3}}\frac{q_f}{w} + \frac{1}{\sqrt{3}}}$$

From (6) and (8), $\frac{q_f}{w} = \frac{F_{fG}}{F_{fL}}$ in equilibrium. Therefore, the Lindahl equilibrium, E, can be found at the tangency point between YY' and the isoquant $\overline{X}_f$. The tangency between the isoquant and YY' reflects the equilibrium condition *(i)* and *(vii)* in Definition 1.

YY' also represents the cost line for sector m. Using (4), (8) and (9), the cost line for sector m is

$$p_m\overline{X}_m = wL_m + q_mG$$

$$= w(\mathcal{L} - a_gG - L_f) + (a_gw - q_f)G = -wL_f - q_fG + w\mathcal{L}$$

whose slope is equal to $\frac{dy_E}{dx_E}|_{EE}^{f}$. Again, the equilibrium condition, $\frac{q_m}{w} = \frac{F_{mG}}{F_{mL}}$, implies that the isoquant $\overline{X}_m$ and YY' are tangent.

Once the equilibrium w and q_i's are determined,[5] the slope of the cost line YY' is given. We can then depict the Lindahl equilibrium to be point E on the efficiency locus in Figure 2. The fact that point E lies on the efficiency locus

5) The wage rate and the Lindahl taxes are determined independent of factor supply. See the factor price equalization result in Khan (1983).

indicates that the Lindahl equilibrium is Pareto optimal;

Lemma 2 *In Lindahl equilibrium, the Samuelson-Kaizuka-Sandmo condition is satisfied.*

Proof. Dividing (9) with w yields

$$\frac{p_f F_{fG}}{w} + \frac{p_m F_{mG}}{w} = a_g \tag{15}$$

We can rewrite this expression using the marginal productivity pricing of labor (6), and obtain the Samuelson-Kaizuka-Sandmo condition

$$\frac{F_{fG}}{F_{fL}} + \frac{F_{mG}}{F_{mL}} = a_g \quad \blacksquare$$

Corresponding to E are the equilibrium output levels X_f^* and X_m^*. Labor allocations for the production of X_f^* and X_m^*, L_f^*, L_m^* and G^*, are measured by the vertical distances from E to AB, AC and BC respectively. The diagram also indicates the amount of Lindahl taxes incurred by each sector, $q_i G$. Since along the BC axis $G = 0$, the vertical distance from point Y, where the cost line YY' intersects BC, to BA indicates the total cost of producing X_f measured in terms of labor. Using the projection on the BC axis, we can see that BY is $\frac{2}{\sqrt{3}}$ times the total cost, Ba is the share of labor cost and aY is the share of Lindahl taxes incurred by sector f, $q_f G$. Similarly, the share of Lindahl taxes incurred by sector m, $q_m G$ is given by Yb.

3.2 Properties of the Efficiency Locus

We have seen that the Lindahl equilibrium lies on the efficiency locus along which the Samuelson-Kaizuka-Sandmo condition is satisfied. In order for the comparative static analysis of the Lindahl equilibrium, it is useful to know the

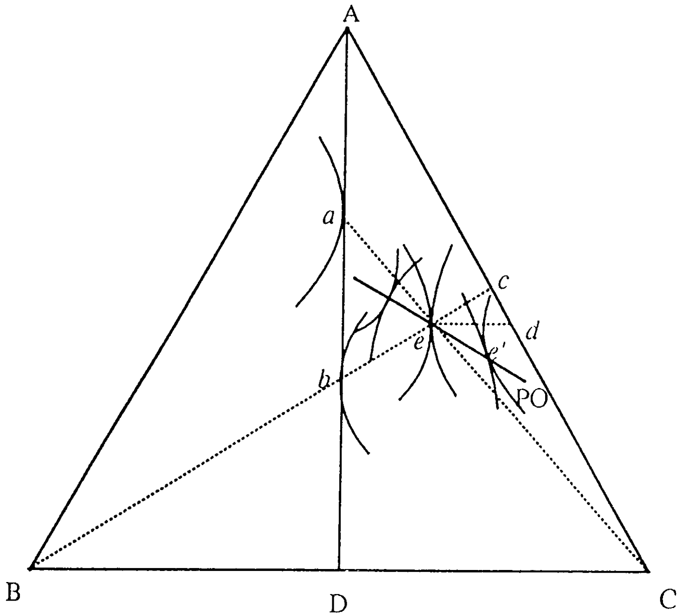

Figure 3: Properties of the Efficient Allocations

shape of the efficiency locus.

With additional assumptions on the relative factor intensity, efficiency locus has the following properties.

Lemma 3 *The efficiency locus lies to the right of AD in Figure 3 when sector m is more public input intensive relative to sector f. The efficiency locus lies to the left of AD when sector f is more public input intensive relative to sector m and it never crosses the line AD.*

Suppose a point of the efficiency locus lies on the line *AD*. Along the line *AD*, $L_f = L_m$ always holds, hence sector *f* and *m* have an identical labor-public input ratio. In the absence of the factor intensity reversals, the efficiency locus necessarily coincides with AD. The two sectors share identical marginal rates of substitution between labor and public input. This also implies that when factor intensities differ between sectors, the efficiency locus will never cross the line AD.

The efficiency locus when sector m is relatively public input intensive is depicted in the figure. Let us define $\gamma_i \equiv \frac{L_i}{G}$. Then we have $\gamma_f > \gamma_m$. Point a where the isoquant for sector m is tangent to the line AD lies above point b where the isoquant for sector f is tangent to. This indicates that when the two isoquants have the same marginal rate of substitution,[6] we can observe $\gamma_f > \gamma_m$.

An immediate implication of the relative factor intensity is that when sector m is more public input intensive, an equilibrium allocation must be located in ADC in which $\gamma_f > \gamma_m$ is always satisfied. If sector f is more public input intensive, an equilibrium allocation is in ABD where $\gamma_f < \gamma_m$ is satisfied.

For further determining the slope of the efficiency locus, an assumption on the relative size of the elasticity of substitution between sectors is necessary. With constant returns to scale technology, the marginal rate of substitution between labor and public inputs, which we denote by $MRS_i \equiv \frac{F_{iG}}{F_{iL}}$, depends solely on the factor proportions, γ_i; as γ_i increases, MRS_i increases.[7] However, whether the efficiency locus is sloped upward or downward depends on the responsiveness of MRS_i as γ_i changes. Such responses are represented by the elasticity of factor substitution, σ_i, which we define to be $\frac{d\gamma_i}{dMRS_i}\frac{MRS_i}{\gamma_i}$.

Lemma 4 *Suppose X_f is increased and X_m is decreased along the efficiency*

6) When isoquants for both sectors are tangent to the horizontal line, $\frac{dx}{dy}|_f = \frac{dx}{dy}|_m = 0$. From (13) and (14), we have $\frac{F_{fG}}{F_{fL}} = \frac{F_{mG}}{F_{mL}} = \frac{a_g}{2}$.
7) Graphically speaking, from (13) and (14}) that the slope of the isoquant $\frac{dx}{dy}|_i$ gets flatter as MRS_i increases.

locus, then

i) γ_f *increases and* γ_m *decreases,*

ii) - a) if $\theta_{fL}\frac{\sigma_f}{\theta_{fG}} < \theta_{mL}\frac{\sigma_m}{\sigma_{mG}}$, G *increases and*

ii) - b) if $\theta_{fL}\frac{\sigma_f}{\theta_{fG}} > \theta_{mL}\frac{\sigma_m}{\sigma_{mG}}$, G *decreases.*

Proof. i) Since the Samuelson-Kaizuka-Sandmo condition, (10), is always satisfied, the sum of marginal rate of substitution between pubic input and labor for each sector is constant. The fact that the higher (lower) marginal rate of substitution, MRS_i, corresponds to higher (lower) labor-public input ratio, γ_i, implies that γ_f and γ_m change in the opposite directions. If MRS_f increases due to an increase in γ_f, MRS_m must decrease so as to satisfy (10), which implies a decrease in γ_m. Conversely, an increase in γ_m also implies a decrease in γ_f. Under constant returns to scale technology, we can write production functions to be

$$X_f = F_f(L_f, G) = G\ f_f(\gamma_f),\ f_f' > 0$$

$$X_m = F_m(L_m, G) = G\ f_m(\gamma_m),\ f_m' > 0$$

Since γ_f and γ_m move in opposite directions, the above equations indicate that an increase in X_f and a decline in X_m implies γ_f to rise and γ_m to decrease.

ii) Dividing (4) by G gives us

$$\gamma_f + \gamma_m + a_g = \frac{\mathcal{L}}{G}$$

This indicates that whether G increases or decreases depends on the magnitude of the changes in γ_f and γ_m. If the size of the increase in γ_f exceeds that of γ_m, we can infer G to decrease (hence the term $-\frac{\mathcal{L}}{G}$ gets smaller).

From (10), we know that $dMRS_f = - dMRS_m$. Hence, what remains to be seen is whether $\frac{d\gamma_f}{dMRS_f}$ is greater or smaller than $\frac{d\gamma_m}{dMRS_m}$. We define the marginal rate of substitution between labor and public input to be $\sigma_i = \frac{d\gamma_i}{dMRS_i}\frac{MRS_i}{\gamma_i}$. We can write

$$\frac{d\gamma_i}{dMRS_i} = \theta_{iL}\frac{\sigma_i}{\sigma_{iG}}$$

where $\theta_{iL} = \frac{wL_i}{p_iX_i}$ and $\theta_{iG} = \frac{q_iG}{p_iX_i}$ We have $\frac{d\gamma_f}{dMRS_f} < \frac{d\gamma_m}{dMRS_m}$ if and only if $\theta_{iL}\frac{\sigma_f}{\theta_{fG}} < \theta_{mL}\frac{\sigma_m}{\theta_{mG}}$, which result in an increase in G. When $\frac{d\gamma_f}{dMRS_f} > \frac{d\gamma_m}{dMRS_m}$, we have $\theta_{fL}\frac{\sigma_f}{\theta_{fG}} > \theta_{mL}\frac{\sigma_m}{\theta_{mG}}$, which leads to a decrease in G. ∎

The efficiency locus drawn in Figure 3 represents the case of *ii) - b)*. As X_f increases and X_m decreases, point e on the efficiency locus moves to e'. As a result, from i) in the lemma, both Bc and Cc rotate clockwise. (This only indicates that e' lies within ceC; the slope of the efficiency locus can be either upward or downward sloping.) The efficiency locus slopes upward if G increases and downward if G decreases. (e' lies within ced if G increases and it lies within deC if G decreases.) Since e is chosen arbitrarily, from ii) in the lemma, the efficiency locus is downward sloping if $\theta_{fL}\frac{\sigma_f}{\theta_{fG}} > \theta_{mL}\frac{\sigma_m}{\theta_{mG}}$, and is upward sloping if $\theta_{fL}\frac{\sigma_f}{\theta_{fG}} < \theta_{mL}\frac{\sigma_m}{\theta_{mG}}$.

4 Comparative Static Analysis

In this subsection we explore some comparative statics results of the Lindahl equilibrium using the diagram. We can confirm the results obtained by the Ricardo-Lindahl model in Khan (1983).

We first examine the relationship between p_i's and factor prices, w^* and q_i^*'s. Under constant returns to scale technology and the absence of joint production, we can write the cost functions for each sector as

$$p_i = wa_{iL}(w, q_f, q_m) + q_i a_{iG}(w, q_f, q_m)$$

where a_{iL} and a_{iG} are the amount of labor and public input required for the production of a unit output. Along with the government's budget constraint, $q_f + q_m = wa_g$, the equilibrium w and q_i are determined solely by p_i's. The decomposition property of the model[8] is now clear. Thus, international trade leads to factor price as well as Lindahl price equalization.

The relationship between the relative output price, $\frac{p_f}{p_m}$, and the relative factor price, $\frac{q_i}{w}$, can be described as follows.

Lemma 5 *A rise in $\frac{p_f}{p_m}$ increases $\frac{q_f}{w}$ and decreases $\frac{q_m}{w}$. A decline in $\frac{p_f}{p_m}$ decreases $\frac{q_f}{w}$ and increases $\frac{q_m}{w}$.*

Proof. Using the price equals unit cost equation

$$p_i = w(a_{iL} + \frac{q_i}{w} a_{iG})$$

where a_{iL} and a_{iG} are the amount of labor and public input required for the production of a unit output, we can write the relative price to be

$$\frac{p_f}{p_m} = \frac{a_{fL} + \frac{q_f}{w} a_{fG}}{a_{mL} + \frac{q_m}{w} a_{mG}}$$

From (9), we have $\frac{q_m}{w} = a_g - \frac{q_f}{w}$. The relative price uniquely determines $\frac{q_f}{w}$, and hence $\frac{q_m}{w}$ through (9), independent of factor endowments. The above equation now takes a form

$$\frac{p_f}{p_m} = \frac{a_{fL} + \frac{q_f}{w} a_{fG}}{a_{mL} + (a_g - \frac{q_f}{w}) a_{mG}} \tag{16}$$

On totally differentiating, we obtain

$$\frac{d(p_f/p_m)}{d(q_f/w)} = \frac{1}{[a_{mL} + (a_g - \frac{q_f}{w}) a_{mG}]^2} [(a'_{fL} + \frac{q_f}{w} a'_{fG} + a_{fG})(a_{mL} + (a_g - \frac{q_f}{w}) a_{mG})$$

8) See Khan (1983).

$$
\begin{aligned}
& - (a_{fL} + \frac{q_f}{w} a_{fG})(a'_{mL} + (a_g - \frac{q_f}{w})a'_{mG} - a_{mG})] \\
= & \frac{1}{[a_{mL} + (a_g - \frac{q_f}{w})a_{mG}]^2} [a_{fG}(a_{mL} + (a_g - \frac{q_f}{w})a_{mG}) \\
& + a_{nG}(a_{fL} + \frac{q_f}{w} a_{fG})] > 0
\end{aligned}
$$

where we have used the cost minimization condition $a'_{fL} + \frac{q_f}{w} a'_{fG} = 0$ and $a'_{mL} + (a_g - \frac{q_f}{w})a'_{mG} = 0$. From (9), $d(\frac{q_m}{w}) =- d(\frac{q_f}{w})$. Hence, $\frac{d(p_f/p_m)}{d(q_m/w)} < 0$. ∎

The relationship between the relative output price, $\frac{p_f}{p_m}$, and the Lindahl tax-wage ratio, $\frac{q_i}{w}$, are represented by PP in the Figure 4 below. This result is in the spirit of the Ricardo-Viner model; in a competitive market, a rise in say p_f while p_m being constant, leads to increase in q_f and w, which in turn reduces q_m because p_m has not changed. We can perceive q_i's to be the factor prices of fictitious factors which are specific to each sector.

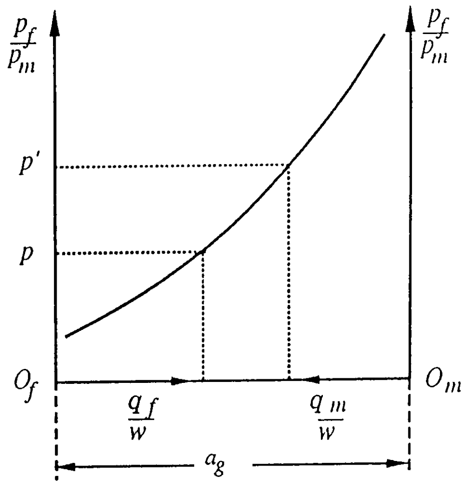

Figure 4: Output Prices and Factor Prices

PP illustrates a positive relationship between $\frac{p_f}{p_m}$ and $\frac{q_f}{w}$, and a negative relationship between $\frac{p_f}{p_m}$ and $\frac{q_m}{w}$. $\frac{q_f}{w}$ is measured on the horizontal axis taking O_f as the origin, while $\frac{q_m}{w}$ is measured from the O_m. From (9), the distance between O_f and O_m is equal to a_g.

Next, we examine the effects of the changes in output prices on output levels of both final goods and public inputs.

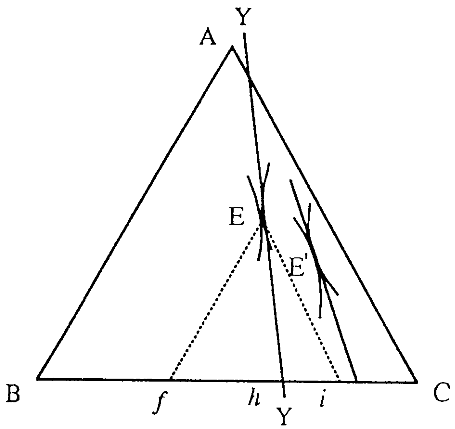

Figure 5: Response to an Increase in p_f/p_m

We can now observe that as the relative output price, $\frac{p_f}{p_m}$, goes up, $\frac{q_f}{w}$ increases which means YY gets flatter and the equilibrium moves to E'. Recall that when sector m is more public input intensive relative to sector f and $\theta_{fL}\frac{\sigma_f}{\theta_{fG}} > \theta_{mL}\frac{\sigma_m}{\theta_{mG}}$, the efficiency locus is downward sloping as shown in the figure. Hence an increase in $\frac{q_f}{w}$ leads to i) an increase in X_f, ii) a decrease in X_m, iii) a decrease in G. The cost share of public inputs increases in sector f while it decreases in sector m. (q_f increases and q_m decreases.)

It should be noted that the well behaved result shown above hinges on the assumption on the relative size of $\theta_{iL}\frac{\sigma_i}{\theta_{iG}}$[9], that is the elasticity adjusted factor intensity, because it determines the direction of the changes in G. When an increase in the relative price of f induces a reduction in G, the output of the labor intensive commodity f increases and the output of the public-input-intensive commodity m decreases. The Rybczynski-like effect due to the change in G is observed here.

Finally, we turn to the effects of changes in the level of labor supply on output levels, it has been shown that an increase in the supply of labor results in a proportional output increases in both sectors and the level of public input.[10]

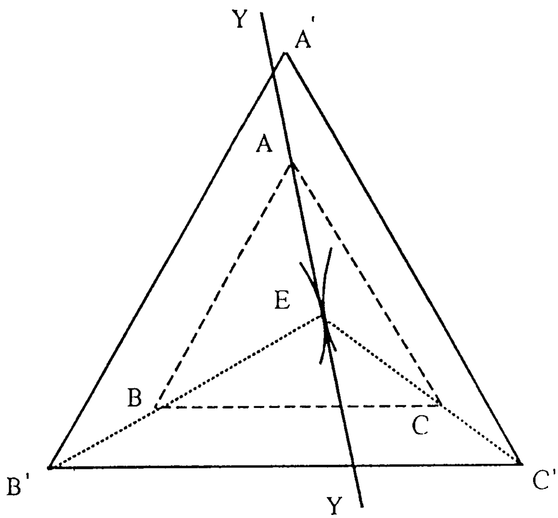

Figure 6: Rybczynski Effect

An increase in $\mathcal{L}$ expands the equilateral triangle from ABC to $A'B'C'$, where $d\mathcal{L}$ is equal to an increase in the height of the triangle. Vectors $B'E$ and

9) See equations (2.15) in Khan (1983).
10) Refer to equation (2.5) in Khan (1983).

$C'E$ reflect new equilibrium output levels of commodity f and m. As a consequence of the factor price equalization, changes in the endowment level does not affect equilibrium factor prices. Therefore, $\mathcal{L}$ does not alter the labor-public input ratio, $\frac{L_i}{G}$; the slopes of $B'E$ and $C'E$ remains the same as those of BE and CE.

5 Concluding Remarks

In this paper, we have examined the Pareto-optimal condition for public input provision and implications for basic trade theorems in the context of a small open economy. Public inputs were introduced to the conventional trade theory of the Ricardo-Viner model. Since the government endogenously determines the optimal level of public inputs according to the Samuelson-Kaizuka-Sandmo condition, the changes in the level of public input turns out to play an important role in determining the direction of changes in outputs. An increase in an output price raises wage and the Lindahl prices for that sector but reduces the Lindahl price for the other sector, which affects the optimal level of public inputs and thus the output levels. Diagramatic analysis presented in this paper brings out such response of public inputs to the changes in output prices.

It should be noted that our analysis was confined to the case of constant returns to scale technology, i.e., the 'unpaid factors of production' type public inputs. This assumption gurantees the transformation curve to be concave and thus the existence of the Lindahl equilibrium, which allows us to generalize conventional trade theorems. When public inputs generate scale economy, Lindahl-tax scheme may no longer be applicable.[11]

11) For further discussion regarding the more general class of technology, see Yukutake (1996).

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