Time-Invariant Linear Filters and Real GDP: A Case of Japan

Takeshi Otsu

Abstract

We compare distortionary effects of time-invariant linear filters used in the economic literature. We investigate the HP filter (Hodrick and Prescott, 1997), the Butterworth filters, and BK filter (Baxter and King, 1999) in filtering real GDP of Japan. We find that the tangent-based Butterworth has the smallest distortion.

1. Introduction

Empirical studies in economics frequently use filtering methods to de-noise time series, extract a trend of time series, and adjust seasonal effects. For example, Kydland and Prescott (1982) use the so-called HP filter developed by Hodrick and Prescott (1997) to investigate the properties of their real business-cycle model. They filter actual data and time series data generated from their model to discuss validity of the model and stylized facts of real business cycle. Another example is concerning seasonal adjustments. Many economic time series exhibit seasonal fluctuations, which obscure underlying economic changes. Researchers apply various statistical models and filtering techniques to implement seasonal adjustment. A comprehensive survey is found in Ghysels and Osborn (2001). The X-12-ARIMA, developed by the U.S. Census of Bureau, is one of the well-known methods frequently used by official governments in many countries. Findley et al. (1998) provide a detailed explanation on the X-12 -ARIMA seasonal adjustment.

Finally, filtering methods are used to estimate potential output, natural rate of unemployment, structural budget deficit or total factor productivity (TFP). It is possible to take advantage of economic theories with functional specifications, as implemented by Congressional Budget Office (2001) of the U.S., if extensive economic database is available and theoretical relationships are stable. In many countries, especially developing countries, we may not have reliable economic data sufficient to estimate theoretical relationships. In such a case, we often rely on filtering methods to extract a trend component, which is supposed to correspond to estimates of unobservable 'potential,' 'natural' or 'structural' variables. Use of the HP filter and its variants is prevailing. De Masi (1997) applies the HP filter to extracting a trend in residuals from a production function, estimates TFP for Japan. de Brouwer (1998) uses it to estimate potential GDP of Australian economy. European Commission (1995) makes use of the HP filter to estimate the cyclically adjusted government budget balances for the member states of the European Union. Most recently, Gerlach and Yiu (2004) estimate output gaps in Asian countries by means of filtering methods.

The HP filter is most frequently used in empirical economics. The seminal paper of Hodrick and Prescott (1997), circulated in 1980 as a working paper, spawns a number of researches that examine the filtering method pros and cons, and that propose various types of filters to decompose economic time series into a trend component and cyclical components. One of the problems discussed in the literature is that the HP filtering induces spurious cyclical effects. That is, the filtered series exhibits periodicity which does not exist in the original series. King and Rebelo (1993) point out that the HP filter alters second moments of time series to a large extent, and claim that stylized facts about periodicity and co-movement of real business cycle would be artifacts because they are based on the second moments. They also study conditions under which the HP filter is optimal when a time series follows ARMA-type stochastic processes. They argue that the conditions are unlikely satisfied. Gerlach and Yiu(2004)find that Asian countries' data give no supportive evidence for the conditions. Reeves et al. (2000) generalize the HP filtering to a multivariate case and propose maximum-likelihood estimation under normality.

Pedersen (2001) argues that the difference in the second moments cannot be taken as evidence of a failure of a specific filter. One of the distortions caused by filtering is the Slutzky effect (Slutzky, 1937). Harvey and Jaeger (1993) and Cogley and Nason (1995) claim that the HP filter has the Slutzky effect to create spurious cycles. What they find is a peak in the power transfer function of the subcomponent of the HP filter. But, this implies that even an ideal filter may cause the spurious effect. Pedersen (2001) claims that the Slutzky effect should be defined as cycles in the power transfer function of the overall, but not subcomponent, filter. Then, no ideal filter to extract targeted periodicity induces the Slutzky effect. According to this definition, the HP filter does cause the Slutzky effect.

Another problem to use the HP filter is how to determine a smoothing parameter, which penalizes variability in the growth component series. Hodrick and Prescott (1997) suggest 1600 for the smoothing parameter for quarterly data. For annual data, the value is typically set to 100 in the empirical literature (Backus and Kehoe, 1992; European Commission, 1995). Pedersen (2001) studies time series of five autoregressive processes and finds that the optimal value, in terms of mean squared errors, ranges from 1007 to 1269 for quarterly sampled series, from 3.73 to 5.03 for annual series, and from 83,200 to 158,800 for monthly series.

The frequency domain analysis reveals that the smoothing parameter is closely related to the periodicity extracted by the filter. Then, if we know what periodicity we should extract from the original series, we could identify what value the smoothing parameter should be. In the empirical studies on the business cycles in the U.S., the business cycles are often considered last between eighteen months and eight years, following Burns and Mitchell (1946). Then, if we could derive the specific relation between the periodicity and the smoothing parameter, we argue what value the parameter should take. Gomez (2001) clarifies this point. It is shown that the HP filter is a special case of the Butterworth digital filter based on the sine function, and that the smoothing parameter is a function of a cut-off frequency, which is determined by the targeted periodicity. Baxter and King (1999) propose a time-invariant band-pass filter to remove certain periodic components. Christiano and Fitzgerald (2003) propose a time-varying band-pass filter.

The purpose of this paper is to study distortionary effects of several time-invariant linear filters frequently used in the economic literature. We measure the distortionary effects with a mean-square-error metric considered in Christiano and Fitzgerald (2003) and Pedersen (2001). We examine a class of Butterwoth digital filters, which incorporate the HP filter as a special case, and a class of finite approximation to the ideal filters, including the Baxter-King filter (Baxter and King, 1999). We find that the tangent-based Butterworth filter gives the best performance when we extract a cyclical component less than eight years from real GDP of Japan. We exclude the band-pass filter developed by Christiano and Fitzgerald (2003), because it is time-varying and left for the ongoing project.

This paper is organized as follows. In the next section, we present a metric to measure filtering distortions. Our argument is based on the mean -square-error criterion. In section 3, we summarize time-invariant linear filters that we investigate here. In section4,we present empirical results, using the annual real GDP of Japan. In the final section, we summarize the findings and discuss the future research.

2. Filtering Distortion

Before discussing filtering distortion, we briefly present notions used here. Suppose we analyze a real-valued stationary stochastic process with finite second moments, y_t $(-\infty < t < \infty)$. The autocovariance generating function is given as

$$g_y(B) = \sum_{t=-\infty}^{\infty} r_t B^t \tag{2.1}$$

where B denotes a backshift operator, and rt gives the autocovariance at lag t. The Fourier transform of rt gives the power spectrum (or the power spectral density function). It can be written with the autocovariance generating function $g_y(b)$ as, at an angular frequency ω,

$$f_y(\omega) = \frac{1}{2\pi} g_y(e^{-iw}), -\pi \leq \omega \leq \pi \tag{2.2}$$

The ‘i’ denotes the imaginary number. Let a time-invariant linear filter

$$h(B) = \sum_{j=-\infty}^{\infty} a_j B^j \tag{2.3}$$

where a_j takes a real number and absolute summable. If l is equal to k, the filter is a symmetric filter. The series filtered with this filter is written as

$$c_t = h(B)y_t = \sum_{j=-\infty}^{\infty} a_j B^j y_t \tag{2.4}$$

The Fourier transform of aj is called the frequency response function (or transfer function) of the filter, which is expressed as $h(e^{-i\omega})$. It can be decomposed as follows:

$$h(e^{-i\omega}) = |h(e^{-i\omega})| \, e^{-i\phi(\omega)} \tag{2.5}$$

The term $|h(e^{-i\omega})|$ is called the gain of the filter, which captures weights that the filter put on various cyclical components. The function $\phi(\omega)$ is the phase (or the phase angle) of the filter, which captures the time shifts that the filter makes in the time series. Then, the spectrum of the filtered series is related to that of yt as follows:

$$f_c(\omega) = |h(e^{-i\omega})|^2 f_y(\omega) \tag{2.6}$$

The squared gain $|h(e^{-i\omega})|^2$ is the power transfer function of the filter, which we denote by $\mathrm{H}(\omega)$.

Suppose we extract frequencies below or above specified cutoff frequency. A low-pass filter (lp) passes frequencies below the cutoff frequency (ω_1). The power transfer function of an ideal low-pass filter satisfies

$$H^*_{lp}(\omega) = \begin{cases} 1, & |\omega| \le \omega_l \\ 0, & |\omega| > \omega_1 \end{cases} \tag{2.7}$$

Then, the frequencies above the cutoff frequency are attenuated. A high-pass filter (hp) passes frequencies above the cutoff frequency, and hp is equal to 1- lp (*complementarity*). The difference of two ideal low-pass filters gives an ideal band-pass filter. Let lp(l) an ideal low-pass filter with a cutoff frequency of ω_1, and lp(u) one with ω_u, and $\omega_1 < \omega_u$. Then, an ideal band-pass filter is computed as bp = lp(u) - lp(l), and its power transfer function satisfies

$$H_{bp}^{*}(\omega) = \begin{cases} 0, & |\omega| < \omega_l \\ 1, & \omega_1 \leq |\omega| \leqq \omega_u \\ 0, & |\omega| > \omega_u \end{cases} \tag{2.8}$$

Suppose we have interest in extracting a certain band of periodicity, say, ranging from one and half a year to eight years. Then, if we use quarterly time-series data, we set ω_u to π/3, and ω_1 to π/16.

In practice, we need to approximate ideal filters in some ways. The approximating filters typically exhibit three types of departure from the ideal filters: leakage, compression, and exacerbation. Leakage captures the effect that the filter passes frequencies which should be suppressed. Compression means that the filter put too small weights on the frequencies to be passed, that is, the frequency response is less than one. Exacerbation is caused by ripples of the filters: a frequency response corresponds to multiple frequencies. Generally speaking, the first two effects can be alleviated by increasing orders of polynomials of the approximating filter. The increase in orders, however, makes the oscillatory behavior of the filter more rapid and does not lessen the size of the ripples. This phenomenon is called the Gibbs phenomenon in filtering theory (Oppenheim and Schafer 1999, pp. 466-468).

These effects create the Slutzky effect, as pointed out by Pedersen (2001, pp. 1085-1090). If the original time series, however, has negligible values of the power spectrum at the frequencies with severe leakage, compression, or exacerbation, the filtering is less distorting. Therefore, we need to take into account the feature of the power spectrum of the time series to measure overall distortionary effects of filters. With the mean-square-error (MSE) criterion, the Wiener-Kolmogorov solution (Whittle, 1983) gives an optimal business cycle filter in the time domain. We can employ the MSE-type criterion to measure the overall distortion. Let us consider the following MSE criterion:

$$MSE = \frac{1}{T}\sum_{t=1}^{T}(\hat{c}_t - c_t^{*})^2 \tag{2.9}$$

Here, c_t^*(t = 1, ..., T) denotes the true cyclical component of the time series, which is obtained by an ideal filter, and c.tis its estimate with some filter. Then,

$$c_t^{*} = h^{*}(B)y_t \tag{2.10}$$

$$\hat{c}_t = \hat{h}(B)y_t \tag{2.11}$$

Thus, we have

$$MSE = \frac{1}{T}\sum_{t=1}^{T}(\hat{h}(B) - h^{*}(B))^2 y_t^2 \tag{2.12}$$

Since it is not possible to define an ideal filter in the time domain with a finite sample, it is convenient to define it in the frequency domain. Therefore, we consider the following frequency-domain analogue:

$$MSE\_f = \int_{-\pi}^{\pi}(\hat{h}(e^{-i\omega}) - h^{*}(e^{-i\omega}))^2 f_y(\omega)d\omega \tag{2.13}$$

This metric is also considered in Christiano and Fitzgerald (2003). Pedersen (2001) uses a similar metric in discrete term, using the power transfer function. Using our notations, it can be written:

$$Q = \sum_{\omega=0}^{\pi} \left| \hat{H}(\omega) - H^{*}(\omega) \right| 2 f_y(\omega) \Delta\omega \tag{2.14}$$

Since the time series to be filtered is real, the symmetry of the power function requires summation over only a half of the frequency domain. The difference from our metric is that it only takes account of the difference in variance of the filtered series, while our metric is directly related to the conventional mean square error criterion and takes account of difference in signs of frequency response between the two filters. In practical experiences, both metrics give qualitatively similar results.

3. Time-Invariant Linear Filters

In this section, we briefly review time-invariant linear filters that are studied here. We focus on the time-invariant linear filter.

$$c_t = \hat{h}(B) y_t = \sum_{j=-k}^{l} \hat{a}_j B^j y_t \tag{3.1}$$

Further, we consider symmetric filters, so that there is no phase shift. Then, the integer l is equal to k. All the filters considered here would have zero phases except for endpoints.

The HP filter is most intensively used in empirical studies of economics. Hodrick and Prescott (1997) suppose that a time series (y_t) is decomposed into the sum of a growth component (g_t) and a cyclical component (c_t). With a sample size of T, they consider the following minimization problem:

$$\underset{\{g_t\}_{t=1}^{T}}{Min} \left\{ \sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=2}^{T-1} [(g_{t+1} - g_t) - (g_t - g_{t-1})]^2 \right\} \tag{3.2}$$

As shown in King and Rebelo (1993), the HP filter for the cyclical component is expressed as

$$\hat{h}_{HP}(B) = \frac{\lambda[1 - B]^2[1 - B^{-1}]^2}{1 + \lambda[1 - B]^2[1 - B^{-1}]^2} \tag{3.3}$$

As Gomez (2001) shows, the HP filter is a special case of the Butterworth digital filters in electrical engineering. The Butterworth filters belong to a class of the Wiener-Kolmogorov solution (Whittle, 1983) to the mean square error minimization problem of a signal extraction. Under conditions of zero phase shifts, complementarity, symmetry, and zero gain at zero frequency, we obtain the Butterworth filter based on the sine function

$$\hat{h}_{BFS}(B) = \frac{\lambda[1 - B]^d[1 - B^{-1}]^d}{1 + \lambda[1 - B]^d[1 - B^{-1}]^d} \tag{3.4}$$

When the parameter d, the order of the filter, is equal to 2, this is equivalent to the HP filter for the cyclical component. Let ω_c a cutoff frequency, at which the gain of the filter is equal to 0.5. Then, we have the following relation:

$$\lambda = [2\sin(\omega_c/2)]^{-2d} \tag{3.5}$$

Therefore, if we could specify the parameter d and the cutoff frequency, we can compute the smoothing parameter λ. Similarly, in addition to the conditions for BFS, if we restricts the gain of the filter is one at a frequency of π, we obtain the Butterworth filter based on the tangent function (BFT), which is studied in Pollock (2000). The filter is expressed as

$$\hat{h}_{BFT}(B) = \frac{\lambda[1 - B]^d[1 - B^{-1}]^d}{[1 + B]^d[1 + B^{-1}]^d + \lambda[1 - B]^d[1 - B^{-1}]^d} \tag{3.6}$$

As shown in Gomez (2001) and Pollock (2000), we have the following relation:

$$\lambda = [\mathrm{tam}(\omega_c/2)]^{-2d} \tag{3.7}$$

The Butterworth filter has the property of maximal flat gain function (Oppenheim and Schafer, 1999, appendix B). Therefore, the exacerbation is expected minimum. As the order parameter d increases, the leakage and the compression are expected smaller. The low-pass filter for a trend is obtained by1minus high pass filter for the cyclical component. The band-pass filter is obtained by the difference of two low-pass filters with different frequencies.

Baxter and King (1999) consider the following minimization problem and obtain a finite symmetric filter to approximate the ideal filter.

$$\min_{\{a_j\}_{j=-k}^{k}} \frac{1}{2\pi}\int_{-\pi}^{\pi} |\delta(\omega)|^2 \, d\omega, \delta(\omega) = h^*(e^{-i\omega}) - \sum_{j=-k}^{k} \mathrm{a}_j e^{-i\omega j} \tag{3.8}$$

The approximate filter has k lags and k leads. This loss function attaches equal weight to the squared approximation errors at different frequencies, whereas the MSE in eq. (2.13), the objective function to derive the Butterworth filters, gives different weights to different frequencies. Solving the minimization problem in eq. (3.8), as explained in Baxter and King (1999, Appendix B), we can obtain the weights of the approximate filter as follows.

$$\hat{\mathrm{a}}_j = \frac{1}{2\pi}\int_{-\pi}^{\pi} h^*(e^{-i\omega})e^{i\omega j} d\omega, \text{ for j} = 1, 2, \ldots k \tag{3.9}$$

Evaluating the integral with some cutoff frequency, ω_c, we have

$$\hat{a}_0 = \frac{\omega_c}{\pi}, \hat{a}_j = \frac{\sin(j\,\omega_c)}{j\,\omega_c}, \text{ for j} = 1, 2, \ldots k \tag{3.10}$$

We call the filter with these weights the finite approximation (FA) filter. Further, Baxter and King (1999) consider a constraint so that the filter has unit weight at the zero frequency, which is equivalent to the filter weights sum to unity. Then, we obtain the so-called Baxter-King (BK) filter.

$$b_0 = \mathrm{a}_0 - \theta, b_j = \mathrm{a}_j - \theta, \text{ for j} = 1, 2, \ldots \mathrm{k} \tag{3.11}$$

$$\theta = \frac{1 - \sum_{j=-k}^{k} \mathrm{a}_j}{2k + 1} \tag{3.12}$$

The band-pass filters are similarly computed as subtracting the coefficient $\mathrm{b_j}$ of a high frequency ω_u and that of a low frequency ω_l .

$$b\_bp_j = b_j(\omega_u) - b_j(\omega_l), \text{ for j} = 1, 2, \ldots k \tag{3.13}$$

4. Empirical Results

We use real GDP data of Japan from NEEDS (Nikkei Economic Electronic Databank System, March 2006). We choose annual real GDP based 68SNA because it is available for the longest periods with a consistent data compiling method. The sample period ranges from 1955 to 2000. The base year is 1990. The sample size is 46. We take logarithm of real GDP data. To compute the metric discussed in the previous section, we compute the power spectrum of the real GDP. We estimate the power spectrum by the Fourier transform of the original series and by the ARMA-type estimating methods that Parzen (1969) and Berk (1974) have proposed. We estimate ARMA model with Box-Jenkins procedure, and select AR (4) model, based on the statistical significance of the parameter estimates, the information criterion, and Ljung-Box Q statistics. **Table 1** shows the estimating result of the ARMA model with the annual real GDP data.

We compute the metric of filtering distortion (eq.2.13) in case that we intend to extract cyclical component less than eight years with annual data. The metric can be computed by using either numerical integration or discrete summation. We use Matlab software to compute the metric. Both computations give the same results as long as the number of grid points of discrete summation is large enough. We use 32,000 equally spaced grid points. To save space, we only report the results of the discrete summation.

First of all, the ad hoc choice of the smoothing parameter value gives rise to great distortion. It becomes clear if we compare the result from the

Table 1 Estimation results of ARMA model

Parameter	Estimate	(Standard Error)	P-value
ϕ_1	1.5061	(0.1246)	[0.000]
ϕ_2	-0.7457	(0.2454)	[0.002]
ϕ_3	0.5357	(0.2404)	[0.026]
ϕ_4	-0.3169	(0.1223)	[0.010]
CONSTANT	0.2741	(0.0662)	[0.000]
Log-Likelihood	113.1420		
Schwartz Bayes Information Criterion	-103.5700		
Q-stat (6lags)	4.3960		0.1110
Q-stat (12lags)	9.6340		0.2917
Q-stat (18lags)	18.3248		0.1924

Model: $y_t = \text{constant} + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \phi_4 y_{t-4} + \varepsilon_t$, $y_t = \log(\text{real GDP}_t)$
ε_t : error term at time t.

Butterworth filter of order two with those from the HP filters. Baxter and King (1999) and Hassler et al. (1994) argue that it would be better to set the smoothing parameter value to ten for annual data. Pedersen (2001) points out the value should be around five for annual data. As shown in **Table 2**, these values give poorer performance than the sine-based Butterworth filter with d=2. Secondly, the FA filters make the worst results. BK filters relatively perform well. These filters do not necessarily give better performance for longer lag length due to the cyclical nature of the sine function. The smallest performance is given by the tangent-based Butterworth filters. Finally, comparing **Table 2** with **Table 3**, we find that these results do not depend on how to estimate the power spectrum of the time series.

5. Discussion

We compare distortions of several time-invariant linear filters. We find the tangent-based Butterworth filters give the best performance. This is because these filters have minimum exacerbation and the gain of one over the passband and zero over the stopband. Some caveats are in order. First of all, we do not know how to specify the order of the tangent-based Butterworth filter. While a greater order reduces leakage and compression, it requires more leads and lags to compute cyclical component at each point of time. Thus, the size of revisions would be large when new information becomes available. Second, we do not consider time-varying filters. Christiano and Fitzgerald (2003) propose time-varying filters within a framework of minimization problem of mean square errors, as studied by Wiener and Kolmogorov. Finally, we need to study other macroeconomic time series before we conclude that the Butterworth filters are generally useful

Table 2 Filtering Distortion (discrete sum): ARMA-Based Spectrum

Filters (restrictions)	MSE_f (eq.2.13) multiplied by 1000
HP (lambda=5)	1.2246
HP (lambda=10)	1.7474
HP (lambda=100)	6.0655
HP (lambda=400)	11.7491
Butterworth (sine: d=2)	0.9588
Butterworth (sine: d=4)	0.4300
Butterworth (sine: d=6)	0.2811
Butterworth (sine: d=8)	0.2094
Butterworth (sine: d=10)	0.1670
Butterworth (tangent: d=2)	0.8046
Butterworth (tangent: d=4)	0.3654
Butterworth (tangent: d=6)	0.2395
Butterworth (tangent: d=8)	0.1785
Butterworth (tangent: d=10)	0.1424
Finite Approximation (3lags)	25.2252
Finite Approximation (4lags)	25.2252
Finite Approximation (5lags)	6.1600
Finite Approximation (6lags)	1.0270
Finite Approximation (7lags)	6.5588
Finite Approximation (8lags)	6.5588
Finite Approximation (9lags)	1.7165
Finite Approximation (10lags)	0.6146
Finite Approximation (11lags)	2.7818
Finite Approximation (12lags)	2.7818
Baxter & King (3lags)	1.9676
Baxter & King (4lags)	1.5872
Baxter & King (5lags)	0.7029
Baxter & King (6lags)	0.7458
Baxter & King (7lags)	1.3460
Baxter & King (8lags)	1.1592
Baxter & King (9lags)	0.4643
Baxter & King (10lags)	0.4882
Baxter & King (11lags)	1.0064
Baxter & King (12lags)	0.9012

Table 3 Filtering Distortion (discrete sum): Fourier-Based Spectrum

Filters (restrictions)	MSE_f (eq.2.13) multiplied by 1000
HP (lambda=5)	7.3695
HP (lambda=10)	8.0854
HP (lambda=100)	16.8013
HP (lambda=400)	21.4295
Butterworth (sine: d=2)	6.1618
Butterworth (sine: d=4)	2.9908
Butterworth (sine: d=6)	1.9142
Butterworth (sine: d=8)	1.3868
Butterworth (sine: d=10)	1.0850
Butterworth (tangent: d=2)	5.2703
Butterworth (tangent: d=4)	2.5319
Butterworth (tangent: d=6)	1.6091
Butterworth (tangent: d=8)	1.1664
Butterworth (tangent: d=10)	0.9148
Finite Approximation (3lags)	24.4042
Finite Approximation (4lags)	24.4042
Finite Approximation (5lags)	8.9449
Finite Approximation (6lags)	3.6856
Finite Approximation (7lags)	6.8680
Finite Approximation (8lags)	6.8680
Finite Approximation (9lags)	3.4679
Finite Approximation (10lags)	2.2299
Finite Approximation (11lags)	3.2362
Finite Approximation (12lags)	3.2362
Baxter & King (3lags)	9.5388
Baxter & King (4lags)	8.1801
Baxter & King (5lags)	4.7242
Baxter & King (6lags)	3.8830
Baxter & King (7lags)	5.0681
Baxter & King (8lags)	4.6256
Baxter & King(9lags)	2.7662
Baxter & King(10lags)	2.3806
Baxter & King(11lags)	3.2609
Baxter & King(12lags)	3.0585

for empirical analysis in economics.

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